ABSTRACT
Consumption patterns have a great impact in consumption of low income population. In fact, the Average Propensity of Consume (ACP) in long term is less than 1 for people who are not poor, but is close or above 1 for poor people. This means that customs or habits formation are different for each social stratum. So, will these consumption patterns change in times of economic crisis? Or stay as such despite economic crisis or not. Apparently, these habits would not change, which means that consumption patterns are deeply ingrained in customs. We propose a dynamic general equilibrium model where the impact of habits is diagnosed through the variable (ϕ) and observe the consumption path over time to demonstrate that these habits are not influenced by exogenous supply shock since consumption would not suffer any changes. The effects of disturbances must be distributed with greater persistence along time.1

JEL: D11, O47, H83

KEYWORDS: Consumption, habits, economic growth, social programs

RESUMEN
Los hábitos de consumo tienen un gran impacto en el consumo de la población con bajos ingresos. De hecho, la Propensión a Consumir Promedio (PCP) a largo plazo es menor a 1 para aquellas personas no pobres, pero cercana a 1 para personas pobres. Esto significa que los hábitos de consumo son diferentes por estratos sociales. Ante ello, ¿los patrones cambiarán ante crisis económicas? o, ¿permanecerán así en el tiempo? Aparentemente no cambiarían ya que los hábitos de consumo se encuentran fuertemente arraigados a las costumbres. Se propone un modelo de equilibrio general dinámico donde el impacto de los hábitos son capturados con la variable (ϕ) por lo que la senda del consumo en el tiempo no sería influenciada por choques de oferta, por lo que los efectos distorsionados deberán ser distribuidos, con mayor persistencia, a lo largo del tiempo.

JEL: D11, O47, H83

PALABRAS CLAVES: Consumo, hábitos, crecimiento económico, programas sociales.
INTRODUCTION

The vulnerable population receives public endowments, which are allocated by the government through social programs in order to relieve this social condition. In the pursuit of efficiency and equity in society, different social programs were analyzed in this working paper, looking out that these public resources are being delivered without observing the habits formation in the vulnerable population or at least not being pointed out in them. This document tries to explain the behavior of consumers with habit formation and how they impact in the Peruvian economy over time, because is highly probable that the government is missing a large amount of monetary resources per year, turning into inefficient programs and harming the public finance.

Habits formation or consumption patterns are highly related to the economic environment and to exogenous supply shocks, which would have a pretty hard distorting component. Basically, consumption patterns are affected by present and past incomes. Thus, the vulnerable population has not only been affected by limited incomes but also by the volatility of economic cycles. Additionally, the limited access of this population to bank loans, as well as to health insurance; pension system; high level education; among others, worsen their economic situation. Therefore, to relieve this social condition, different governments have supplied public transfers, both monetary and non-monetary goods and services.

To know what the behavior of pattern consumption would be, regions as San Martin, Ayacucho and Cajamarca were chosen to observe the composition of their consumption and which their principal types of expense are. Was analyzed the average consumption propensity of their population, finding interesting results. It should be noted that principal motivations to analyze these regions are three: i) the high rate of poverty; ii) quantity of endowments received by the government through social programs; and iii) large amount of natural resources.

Therefore, the principal goal of this document is to diagnose the behavior of household where there are habits formation, and observe the path of consumption over the time and if it suffers exogenous disturbances to affect these public endowments delivered by government directly. Also, the hypothesis of this document is that these social programs are being considered inside of the methodology of the consumption patterns and these methodological technics are impacting positively to vulnerable income of population. So, the importance of this paper is to focus social programs better in future to accomplish their targets and increase vulnerable income of population and set poverty condition off.

This document is organized as follows. Section 2 presents the empirical studies along with a brief literature review and methodology used to demonstrate the consumption patterns. Section 3 discuss the inter-temporal consumption model among different studies, where they show how this model explains the characteristics of consumers. Section 4 describes the data used in accordance with the model and the section 5 presents the main results against evidences about the parameter $\phi$ which measure the habits formation. The conclusions depend on the significance of this parameter in the dynamic general equilibrium, providing suggestions for future research.

REVIEW OF THE LITERATURE AND STATISTICAL DATA

Different authors have offered diverse explanations about consumption patterns and how the future decisions would have affected. These types of models related to inter-temporal consumption have been used to explain many fields not only in the economic theory, but also in the financial sector. For example, Dai & Grishchenko (2011) had estimated a consumption habit based on asset pricing through application of a stochastic model. In this document they found that stochastic shocks persistent in expected returns.

Also, there have been many methodologies related to models of heterogeneous agent. For example, Carrasco et al (2002) had demonstrated that there was a relationship between current and past consumption. They took a survey of Spanish household doing a panel-data methodology to demonstrate the presence of habit formation in their results and confirmed the importance of the “fixed effects” in this type of methodologies. The main result was residual component did not have random behavior therefore did not affect to inter-temporal consumption decisions.

In fact, the work by Chetty & Szeidl (2016) notably contributed to this discussion. They have showed how the idiosyncratic affect in the decisions of population would be related
with customs and culture; and would produce habits of consumption that are accumulated in time which were determined by weighted average of the past consumption of consumers. So, their findings are attributed to consumption commitment that can be explained for habits of consumption, being extremely smooth and sensitive over time.

Nonetheless, the habits of consumption have been related with incomes of population too. For example, Seckin (2000) found that the path of consumption not only depends of permanent incomes but also of the past consumption. If habits of consumption are so strong in population, the effect of the uncertainty into incomes over current consumption is very low.

More recent, formalized approach was written Byun (2013) who showed some evidence that there is a relationship between habit formations; demographic characteristics; and job status. The correlation between them was worked with no-lineal models applying Dynan’s parametric estimation getting interesting results. Other studies argued that interpersonal comparison has a strong relevance in habit formation as argued Drechsel – Grau & Schmid (2013), whose work demonstrated that reactions of consumption are leaded by perceptions as envy to be rich.

Therefore, the severe constraint in incomes affects to households in poverty condition. So, these families have always taken important and difficult decisions in their consumptions day by day, with incomplete information getting from market and where they interact with a dynamic inconsistence from government. If social programs did not consider behavioral problems from poor population, whose roots have a strong component of habits formation, the individual position in compare with the society possibly wouldn’t change.

Next, the government has initiated different social programs since many year, which being focused to alleviate and overcome current social poor condition. So, poor population have received many assistance and supports actions to improve their social well-being and economic situation through pecuniary endowments and non-pecuniary. Nowadays, the government allocates public funds to social investment to generate new opportunities to recipients to leave from this situation with their own means, through more education, ongoing training, labor experience, etc.

At 2016, the National Statistical Bureau (INEI Spanish acronym) had published that 20.7% of Peruvians were belong poor population which decreased 28.4 pts. since last ten years. In same year, the extreme poor population had been belong 3.8% which decreased 10 pts. in same period. If this analyze is divided by region, the highland region had showed more Peruvians in poor condition (31.7%) while in the coast region much less (12.8%) (see Table 1).

<table>
<thead>
<tr>
<th>Table 1. Peru: Population in poor condition, by region, 2006 – 2016 (% respect to total population)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Urban</td>
</tr>
<tr>
<td>Rural</td>
</tr>
<tr>
<td>Region</td>
</tr>
<tr>
<td>Coast</td>
</tr>
<tr>
<td>Highland</td>
</tr>
<tr>
<td>Rainforest</td>
</tr>
<tr>
<td>Source: INEI</td>
</tr>
</tbody>
</table>

It should be noted that 32.4% of all Peruvian living in conditions of non-poverty, at least one member of the household had received endowments from socials program, while from all Peruvian in poverty condition were 59.2%.

Focusing on regions, San Martin suffered an increase in the number of households that had received these endowments, from 59.6% to 72.9% (2007 to 2016), compared to Cajamarca and Ayacucho.
At same year, the government had allocated public expenses for social programs near S/. 69.2 thousands millions of soles from public budget (13.8% of GDP), basically focusing in two main activities: health / education (55.8% of total expense); and poverty, extreme poverty and social inclusion programs (16.3%). From this last amount, the government had allocated near 5.3% in three main programs: Qali Warma (2%); JUNTOS (1.4%); and Pension 65 (1.2%), respectively (see Table 2).

Table 2.

<p>| Peru: Budget allocated for social program, 2007 – 2016 (% respect to total population) |
|---------------------------------|-----|-----|-----|-----|-----|</p>
<table>
<thead>
<tr>
<th>Total</th>
<th>2007</th>
<th>2010</th>
<th>2013</th>
<th>2016</th>
<th>Var. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>137</td>
<td>141</td>
<td>230</td>
<td>897</td>
<td>5.55%</td>
</tr>
<tr>
<td>Health</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JUNTOS</td>
<td>328</td>
<td>533</td>
<td>1 215</td>
<td>1 586</td>
<td>3.84%</td>
</tr>
<tr>
<td>Pension 65</td>
<td>497</td>
<td>613</td>
<td>891</td>
<td>979</td>
<td>0.97%</td>
</tr>
<tr>
<td>Cuna mas</td>
<td>-</td>
<td>1</td>
<td>451</td>
<td>801</td>
<td>800.00%</td>
</tr>
<tr>
<td>Qali Warma</td>
<td>55</td>
<td>73</td>
<td>181</td>
<td>334</td>
<td>5.07%</td>
</tr>
</tbody>
</table>

Source: INEI

To know what the behavior of consumption patterns and its impact in the Peruvian consumption would be, we have analyzed the consumption expenditures of Peruvians households. The main source of information has been provided by National Survey of Households (ENAH0 Spanish acronym) from INEI, which supplied statistical data from 2004 to 2017. In addition, to narrow our analysis, we chose only three regions: Ayacucho, Cajamarca y San Martin. We must point out that our main motivation to diagnose these three regional governments were three main characteristics: i) high poverty rate; ii) huge allocations for social programs; and iii) exploitation of natural resources.

Following to analyze this data, in 2016 the level of poverty in Cajamarca was located in 50% (71% at 2007); Ayacucho stood at 36.1% (71% at 2007), which belonged to the first two regional groups with a high rate of population living in poverty. San Martin stood at 24.7% (59% at 2007) with respect to total participation, which belonged to the third group of regional government with a high level of poverty. It should be noted that San Martin has not received huge amounts of public revenues from the central government for exploitation of natural resources; against Cajamarca position that has received enormous amounts by Canon (obtaining public incomes through the exploitation of natural resource). Also, Ayacucho has not received large amounts of public revenues (Canon or others) despite having a large population in conditions of poverty.

The effects of income over consumption have had a huge connotation in the economic theory. The low growth of the GDP influenced by different negative aggregate supply shocks (oil and mineral prices, etc.) or from aggregate demand (increasing of the tax, fiscal deficit, etc.), have affected the evolution of income over time. Therefore, the consumption behavior is a relevance variable for different countries because it has always been modified by economic policies over time whose goal has always been maintain the social well – being. Muellbauer & Lattimore (1994) noted that consumption expenditures have meant approximately 50% and 70% over total expenditures over a big number of economies, having a high correlation in compare to macroeconomic aggregates, also being the principal source of economic growth in the world. Peru has not been the exception. It has shown the same kinds of consumption expenditures since 2004 to 2016, without significant changes at different types of geographic zones.
Additionally, different works have tried to explain what main characteristics of the consumer behavior were, arising different types of economic theories. One of them is based on relative incomes (Duesenberry, 1949), whose hypothesis is a subjective behavior of the consumer related to beliefs, customs, preferences and habits, which have become ingrained during the life of consumer. These habits are reflected in the average propensity to consume (APC). This variable is defined as a proportion of income (between 0 and 1, or more in different cases) that is intended to be consumed.

To construct this APC that has an “S” form, we have gotten as part of incomes: monetary incomes; salaries for principal and secondary activities; rents for principal and secondary independent activities; others incomes; national and foreign remittances. And as part of expenditures: foods; clothes; rental housing and fuel; furniture and equipment, housing maintenance; health; communications and transporting; recreation; others. The period of analysis have been since 2004 until 2016. Approximately, the Peruvian APC is concentrated in 1 (see Figure 1).

### Table 3.

<table>
<thead>
<tr>
<th></th>
<th>Coast (%)</th>
<th>Highland (%)</th>
<th>Rain forest (%)</th>
<th>Lima (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foods</td>
<td>52.7</td>
<td>51.1</td>
<td>53.8</td>
<td>49.4</td>
</tr>
<tr>
<td>Fuel, electricity, water, rent</td>
<td>12.5</td>
<td>11.4</td>
<td>9.7</td>
<td>17.3</td>
</tr>
<tr>
<td>Furniture &amp; equipment</td>
<td>6.5</td>
<td>7.0</td>
<td>7.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Health</td>
<td>7.9</td>
<td>7.7</td>
<td>8.6</td>
<td>8.6</td>
</tr>
<tr>
<td>Communications &amp; transporting</td>
<td>12.4</td>
<td>12.5</td>
<td>11.0</td>
<td>13.5</td>
</tr>
<tr>
<td>Clothes</td>
<td>8.0</td>
<td>10.4</td>
<td>9.3</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Source: INEI

### Table 4.

<table>
<thead>
<tr>
<th></th>
<th>Coast (%)</th>
<th>Highland (%)</th>
<th>Rain forest (%)</th>
<th>Lima (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foods</td>
<td>50.8</td>
<td>48.0</td>
<td>48.2</td>
<td>44.2</td>
</tr>
<tr>
<td>Fuel, Electricity, Water, Rent</td>
<td>12.2</td>
<td>12.5</td>
<td>11.3</td>
<td>18.8</td>
</tr>
<tr>
<td>Furniture &amp; equipment</td>
<td>6.9</td>
<td>6.5</td>
<td>8.0</td>
<td>7.4</td>
</tr>
<tr>
<td>Health</td>
<td>8.1</td>
<td>8.2</td>
<td>7.9</td>
<td>8.6</td>
</tr>
<tr>
<td>Communications &amp; transporting</td>
<td>13.5</td>
<td>14.5</td>
<td>15.4</td>
<td>14.7</td>
</tr>
<tr>
<td>Clothes</td>
<td>8.5</td>
<td>10.2</td>
<td>9.3</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Source: INEI
But, if we display whole population by quartiles, we would see that the first one (lower incomes or Q1) of the APC is approximately 1, but some observations are above 1 (between 2 or 3) as ACP. This means that some population exceeds its consumption with respect to its income. Nevertheless, in the fourth one (higher incomes), APC is well below 1. This means that these consumers spend less and have more capacity to save; as long as consumers have more incomes than expenses; or take loans. But, does it happen in every economic cycle? or, do they spend less on economic crisis or economic “boom”?

To respond these questions, we have taken the Peruvian economy as a sample to analyze different periods. The first period was been taken since 2004 to 2007, denominated “Peruvian economic pre-crisis”. Which was a good period where the mineral prices had reach the best performance and it could observe a narrow area in the “S” of ACP, meaning an ACP equivalent to 1 approximately, for all population. The second one was taken since 2008 to 2011, called “Peruvian economic crisis”; where the whole world got inside financial crisis due to the Lehman Brothers bankruptcy. The last one was taken since 2012 to 2016 named “Peruvian economic post-crisis”; where different economies began to emerge and therefore the “S” of ACP was more scatter, but in same way, the ACP is equivalent to 1 approximately, for all population (see Figure 3).
Therefore, consumer habits have a strong component in the decision of the consumer. In detail, there was a big difference when it was shown by periods, habits did not change radically before and after the economic crisis. But, the ACP had a big difference when it was shown in quartiles, where the poor population consumed above their income (ACP> 1), while the rich population consumed below their income (ACP <1), where the capacity of savings is greater.

**METHODOLOGY: AN INTER-TEMPORAL CONSUMPTION MODEL**

In this section, we present a methodological discussion of following model. We has evaluated the impact of consumption habits that are introduced in the individual consumption of each person. These habits would be understood as a cost of adjustment when customs, which related to the geographic influences in the population, are of great importance in current consumption. Therefore, we have considered a utility function which has a present consumption and its past consumption. This has been discounted by a proportion of past consumption (Duesenberry (1949); Constantinides (1990); and Schmitt – Grohé y Uribe (2008)) (see equation 1).

\[
\text{Equation 1: } (C_t - \phi C_{t-1})
\]

Also, the utility function form depends on consumption and labor. We assume that there are many consumers whose utilities function have been discounted by value (see equation 2).

\[
\text{Equation 2: } U(\cdot) = \sum_{t=0}^{\infty} \beta^t [\gamma \log(C_t - \phi C_{t-1}) + (1 - \gamma) \log(1 - L_t)]
\]

Likewise, these consumers receive endowments related to the social programs. Baxter & King (1993) suggested introducing a direct public expenditure into the utility function (see equation 3), that is, the consumers not only have a private consume but also public consume and its participation in the utility function depends on the elasticity of each consumption.
Equation 3:
\[
\begin{align*}
\varphi(\cdot) = e^\sum_{p} \left[ \log\left( w^p \cdot c^p_{t-1} + (1-w) c^p_{g,t-1} \right) \right] \left( 1 + \phi \right) \left( 1 - \phi \right)
\end{align*}
\]
Then, consumption depends on a value of substitution that measures the sensitivity of consumption between private goods and public goods and their multipliers: defined as a weighted average; while defined as a tradeoff between consumption and labor. With the preferences defined, the budget constraint is subject to equality between income, consumption and savings (see equation 4).

Equation 4:
\[
\frac{1}{(1 + \tau)^k}(1 - \tau) \left(K_{t+1} - K_t\right) = (1 - \tau)\left(\delta_t + (1 - \tau)^k(\delta_t - \delta)K_t + \tau\right)
\]
In equation 4, we can see how consumption is charged by a tax in same way income coming from labor; and rent to capital assets offered. The consumer receives a public transfer of social program. By making, with the Euler equation, the optimal consumption path can be traced over time (equation 5) where four periods of consumption are observed, assuming that consumer will have incentives to maintain consumption over time, thus avoiding effects volatiles.

Equation 5
\[
\begin{align*}
\frac{C^i_{p,t} - 1 \cdot w^{i}_{p,t} + (1 - w) c^{i}_{g,t-1}}{[wC^i_{p,t} + (1-w) c^{i}_{g,t-1}]^{1+\frac{1}{\psi}}} - \phi \frac{wC^i_{p,t} + (1-w) c^{i}_{g,t-1}}{[wC^i_{p,t} + (1-w) c^{i}_{g,t-1}]^{1+\frac{1}{\psi}}} - \beta \frac{wC^i_{p,t} + (1-w) c^{i}_{g,t-1}}{[wC^i_{p,t} + (1-w) c^{i}_{g,t-1}]^{1+\frac{1}{\psi}}} - \frac{wC^i_{p,t} + (1-w) c^{i}_{g,t-1}}{[wC^i_{p,t} + (1-w) c^{i}_{g,t-1}]^{1+\frac{1}{\psi}}}
\end{align*}
\]

The firms will have to optimize their productive factors. We assume competitive market so the production function has a functional shape as following equation (Barro, 1990):

Equation 6:
\[
Y_t = \varepsilon_t \cdot \tau A_t \cdot K^{\alpha} G_t^{1-\alpha}
\]
Thus, the firm adopts an optimal combination between private capital and public expenditure considered to produce private goods. In addition, we assume that social programs are excludable and rival, so they can only be provided by government. The public revenues come from tax collection to allow offering social programs by the government to vulnerable population with required quantity in time. We should point out that public goods follow a stochastic process reflected in , which has a stationary behavior in time, in average and variance; and following an autoregressive process in first order whose errors have random effects on the consumption of public goods due to exogenous shocks (see equation 6).

To develop this model, we assume a competitive equilibrium to determine the endogenous and exogenous variables. With this assumption, we would see the path of consumption in time and we could introduce random effects on it to recognize how much affect it. That is, how much will increase or decrease the inter-temporal rate of consumption if we consider consumption habits (through the provision of social programs over time) for getting dynamic effects on the path of consumption.
MAIN RESULTS

After observing the main results in the intertemporal model related to the consumption evolution in the time, we are going to display the understanding of effects about habits formation on Peruvian consumer decisions. For that, the model showed the parameter which captures the behavior of these habits; and executing it, we could observe that this parameter would require to be between 80% - 95%, that is, the habits of consumption had a great influence on the current consumption.

In the Figure 4 we can see that the solid curve represents the consumption time series that initiate in period 1. So, to get the steady state over time, the consumption ought to be reaching its maximum levels between 14 and 16 years, approximately, as long as the parameter is located in 85%. If the parameter is located above or below that this percentage, then consumption increase quickly or smoothly, respectively. Our findings imply that attempt to map results into habit formation must be reconsidered in the social programs supplied by the government.

In the same way, in previous section we could observe the ACP as a principal indicator that can show the habits formation which represents a strong component in the decision of Peruvian consumer. This component was composed by different consumes (food, health, and others) and incomes (salaries, incomes for different activities, and others one); so, this ACP is close or above to 1 if this people is considered poverty population; and below 1 for non-poverty population.

Habits of consumption are resistant to changes, so it is assumed that consumption should be stable path over time. Therefore, if there is an exogenous shock in the Peruvian economy, then consumption should not undergo any change and these effects of disturbances should distribute with greater persistence over time.

It should be noted that in our model the public consumption was introduced in utility function, so the possibility that we have a crowding-out with private consumption is bigger one.

Figure 4
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The impact of habits formation over social programs to reduce the poverty in Peru

**APPENDIX**

**HOUSEHOLDS**

The consumption optimization:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log \left( \frac{wC_{p,t}^\eta + (1 - w)C_{g,t}^\eta}{\left( wC_{p,t}^\eta + (1 - w)C_{g,t}^\eta \right)^{\frac{1}{\gamma}}} - \phi \left( wC_{p,t-1}^\eta + (1 - w)C_{g,t-1}^\eta \right)^{\frac{1}{\gamma}} \right) + (1 - \gamma) \log(1 - \lambda_t - (1 - \tau^t_E)(C_t - \phi C_{t-1} - K_{t+1} - K_t - (1 - \tau^t_E)W_t L_t - (1 - \tau^t_F)(R_{t-\delta} - \delta)K_t - T_t) \right]
\]

First orders conditions:

\[
\frac{\partial L}{\partial C} = \beta^t \left[ \frac{\gamma wC_{p,t}^{\eta^{-1}} \left( wC_{p,t}^\eta + (1 - w)C_{g,t}^\eta \right)^{\frac{1}{\gamma}} - \phi \left( wC_{p,t-1}^\eta + (1 - w)C_{g,t-1}^\eta \right)^{\frac{1}{\gamma}}}{\left( wC_{p,t}^\eta + (1 - w)C_{g,t}^\eta \right)^{\frac{1}{\gamma}}} - \lambda_t (1 + \tau^t_E) \right] = 0
\]

\[
\frac{\partial L}{\partial K} = \lambda_t \beta^t (1 - \tau^t_F)(R_{t-\delta} + 1) - \lambda_{t-1} \beta^{t-1} = 0
\]

Inter-temporal optimal consumption: Euler equation

\[
\frac{\lambda_{t-1}}{\lambda_t} = \beta (1 - \tau^t_F)(R_{t-\delta} + 1)
\]

Resolving de \( \lambda_t \) de \( \frac{\partial L}{\partial C} \):

\[
\lambda_t = \left( 1 + \tau^t_F \right)^{-1} \left[ \frac{\phi C_{p,t-1}^{\eta^{-1}} \left( wC_{p,t}^\eta + (1 - w)C_{g,t}^\eta \right)^{\frac{1}{\gamma}} - \phi \left( wC_{p,t-1}^\eta + (1 - w)C_{g,t-1}^\eta \right)^{\frac{1}{\gamma}}}{\left( wC_{p,t+1}^\eta + (1 - w)C_{g,t+1}^\eta \right)^{\frac{1}{\gamma}}} - \beta \left( \phi C_{p,t-1}^{\eta^{-1}} \left( wC_{p,t}^\eta + (1 - w)C_{g,t}^\eta \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \right] \frac{1}{\left( wC_{p,t+1}^\eta + (1 - w)C_{g,t+1}^\eta \right)^{\frac{1}{\gamma}} - \phi \left( wC_{p,t+1}^\eta + (1 - w)C_{g,t+1}^\eta \right)^{\frac{1}{\gamma}}}
\]

**FIRMS**

The firm should maximize its benefits through the optimization from \( \pi_t \) and \( R_t \). The consumption of public goods and public services is defined by firm function, which has a random variable that absorbs the supply shocks.

\[
\pi_t = \epsilon_t \tau A_t K_t^{\alpha - 1} G_t^{1-\alpha} - R_t K_t
\]

\[
\frac{\partial \pi_t}{\partial K_t} = 0: \quad R_t = \alpha \epsilon_t \tau A_t K_t^{\alpha - 1} G_t^{1-\alpha}
\]

\[\text{... (2)}\]
EQUILIBRIUM

Equation (1) and (2):

\[
\begin{align*}
\frac{C_t^{\pi - 1} \{ wC_t^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}}{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}} \left[ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right]^{\eta} - \phi \left( wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right) \right] \\
\frac{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}}{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}} \left[ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right]^{\eta} - \phi \left( wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right) \right] \\
\frac{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}}{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}} \left[ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right]^{\eta} - \phi \left( wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right) \right] \\
\frac{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}}{\phi C_t^{\pi - 1} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{1 - \eta} \left\{ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right\}^{\eta}} \left[ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right]^{\eta} - \phi \left( wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right) \right] \\
= \beta \frac{(1 + \tau^{\xi})}{(1 + \tau^{\xi - 1})} \left( 1 - \tau^{\xi} \right) (\alpha \varepsilon_t \tau A_t) \left[ wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right]^{\eta} - \phi \left( wC_{t-1}^{\pi-1} + (1 - w)C_{t-1}^{\pi-1} \right) \right] + 1
\end{align*}
\]

SPECIFYING THE EQUATIONS OF THE MODEL

1. Euler equation: \( \left( \frac{C_t^{\pi - 1}}{C_t^{\pi - 1}} \right)^{\eta} = \frac{(1 + \tau^{\xi - 1})}{(1 + \tau^{\xi - 1})} \beta (1 - \tau^{\xi}) (R_t - \delta) + 1 \)

2. Kapital price: \( R_t = \alpha \varepsilon_t \tau A_t \left( K_t^{\pi - 1} G_t^{1-\alpha} \right) \)

3. Stochastic process: \( \varepsilon_t = \theta + \rho \varepsilon_{t-1} + \mu_c \)

4. Firm function: \( Y_t = \varepsilon_t \tau A_t K_t^{\pi - 1} G_t^{1-\alpha} + BY_t \)

5. Investment: \( I_t = Y_t - C_t \)

6. Budget constraint: \( k_{t+1} = (1 - \tau^{\xi}) W_t L_t + [(1 - \tau^{\xi}) (R_t - \delta) + 1] K_t + T_t - (1 + \tau^{\xi}) (C_t - \phi C_{t-1}) \)

STEADY STATE:

1. Consumption: \( C_{ss} = \frac{(1 - \tau^{\xi})}{(1 + \tau^{\xi}) (1 - \phi)} (1 + \tau^{\xi}) (1 - \phi) K + \frac{1}{(1 + \tau^{\xi})(1 - \phi)} T \)

2. Kapital: \( K_{ss} = \frac{\frac{\beta}{(1 - \rho)(1 + \tau^{\xi})(1 - \phi)}^{\frac{1}{1 - \alpha}}}{(1 + \tau^{\xi})(1 + \tau^{\xi})(1 - \phi)^{\frac{1}{1 - \alpha}}} \)

3. Stochastic process: \( \varepsilon_{ss} = \frac{\theta}{(1 - \rho)} \)

4. Firm function: \( Y_{ss} = \varepsilon_{ss} \tau A K_{ss} G^{1-\alpha} \)
LOG – LINEAL METHOD BY TAYLOR SERIES:

\[ f(C_{x,t}) + \frac{f'(C_{x,t-1})}{1!} (C_{t-1} - C_{x,t}) \]

Log to Euler equation:

\[ \log C_t = \log C_{t-1} + \frac{\log(1 + \tau^e_{t-1})}{\eta} - \frac{\log(\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(R_t - \delta) + 1)}{\eta} \]

Moving the price of kapital \((R_t)\) in the function of firm where is introduced budget constraint in \(I\). If \(f'(C_{t-1})\) we get:

\[ f'(C_{t-1}) = \frac{1}{C_{t-1}} \left[ \frac{\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(\alpha - 1) \approx \epsilon \alpha A K^{x_2 - 2} G^{1 - \epsilon}}{[\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(R_t - \delta) + 1]\eta} \right] \]

The path of the consumption is:

\[ c_t = \left( \frac{(1 + \tau^e_{t-1})C^0_t}{[\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(R_t - \delta) + 1]} \right)^{\frac{1}{\epsilon}} + \left( \frac{1}{C_{x,t-1}} \left[ \frac{\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(\alpha - 1) \approx \epsilon \alpha A K^{x_2 - 2} G^{1 - \epsilon}}{[\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(R_t - \delta) + 1]\eta} \right] \right) - \left( \frac{1}{C_{x,t-1}} \left[ \frac{[\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(\alpha - 1) \approx \epsilon \alpha A K^{x_2 - 2} G^{1 - \epsilon}}{[\beta(1 - \tau^e_{t})(1 + \tau^e_{t})(R_t - \delta) + 1]\eta} \right] \right) c_{x,t} \]

OBTAINING THE PARAMETERS:

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